Kinetic compressive sensing:
improving image reconstruction and parametric maps

M. Scipioni[1,2], N. Fuin[2,4], M.F. Santarelli[3], L. Landini[1], O.A. Catalano[2,4], C. Catana[2,4], D.N. Greve[2,4], J.C. Price[2,4], and S. Pedemonte[2,4,5]

(1) *Department of Information Engineering, University of Pisa, Pisa, Italy*
(2) *Athinoula A. Martinos Center for Biomedical Imaging, Boston, MA, USA*
(3) Institute of Clinical Physiology, CNR, Pisa, Italy
(4) *Harvard Medical School, Boston, MA, USA*
(5) *Center for Clinical Data Science, MGH, Boston, MA, USA*

Background

Parametric images provide insight into the spatial distribution of physiological parameters, but they are often extremely noisy, due to *low SNR of tomographic data*. **Direct estimation of maps from projections** (1) allows accurate noise modeling, improving the results of post-reconstruction fitting.
We propose a method, which we name **kinetic compressive sensing (KCS)**, based on a *hierarchical Bayesian model* and on a novel reconstruction algorithm, that encodes **sparsity of kinetic parameters**.
The parametric maps are reconstructed by *maximizing the joint probability of all unknown parameters* in the graph in ***Fig.1*** using an **iterated conditional modes (ICM)** approach (3).

Hierarchical Bayesian Model

The model has three key components:

(1) the **model of the acquisition system** consists of the ordinary Poisson model, incorporating all effects of attenuation, scatter and randoms;

(2) the **kinetic model** encodes the assumption that the voxel intensities are noisy realizations of a *hidden dynamic process*, modeled using a *multi-compartmental model.*

(3) a **sparsity-inducing prior** distribution of the kinetic parameters is introduced as a *Markov Random Field (MRF) with Smooth L1-norm cost function* (6).

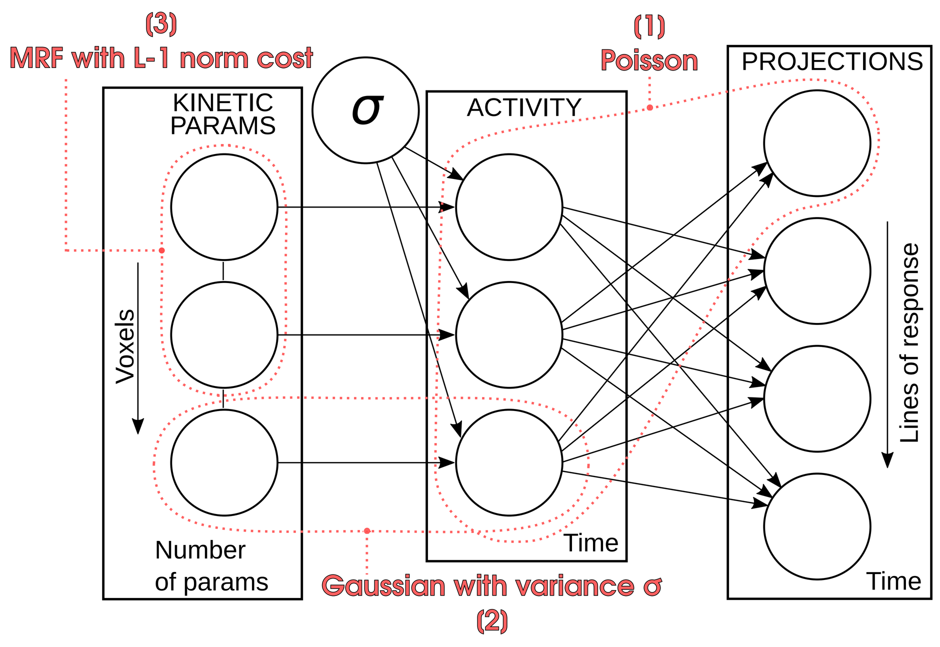
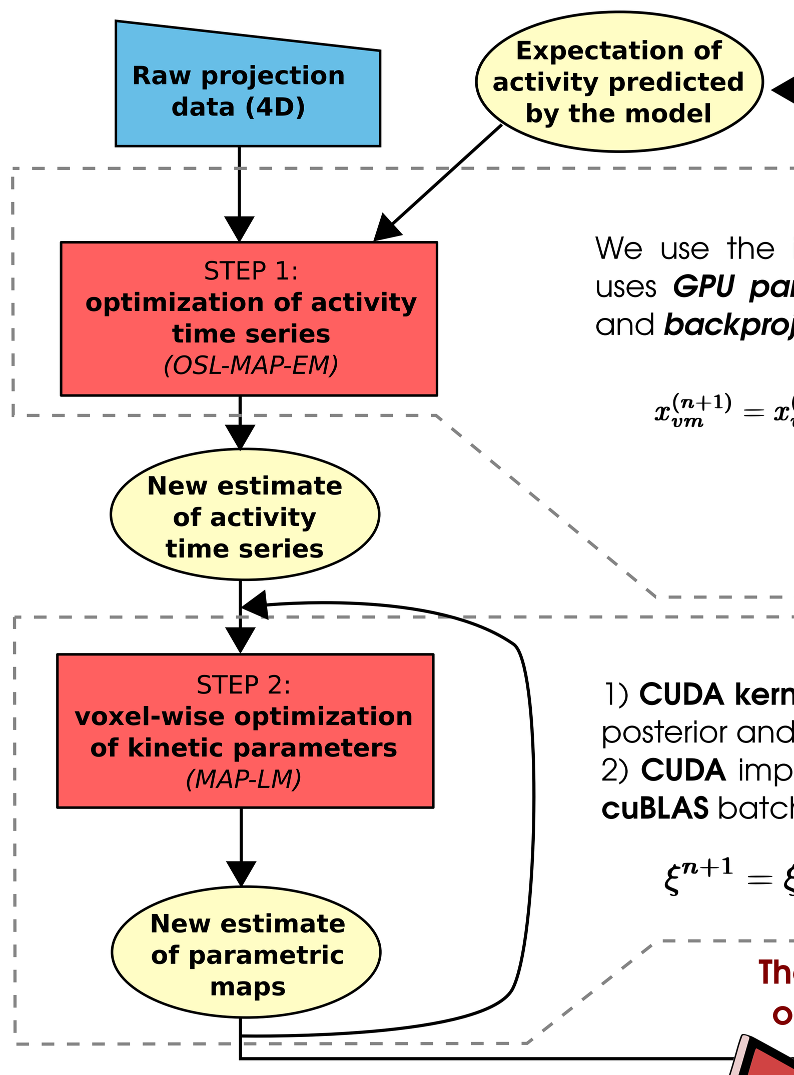


Fig. 1

Algorithm workflow

The algorithm consists of **alternating** the **optimization** of the activity time series and of the kinetic parameters:
1) given the kinetic parameters: ***one-step-late maximum a-posteriori expectation-maximization*** (OSL-MAP-EM) **(4)**
2) given the activity time series: ***maximum-a-posteriori Levenberg-Marquardt*** (MAP-LM) optimization



STEP 1

We use the in-house developed ***Occiput.io*** (5) toolbox, which uses ***GPU parallel computation*** for the operations of ***projection*** and ***backprojection*** involved in the ***OSL-MAP-EM step*** (4):

$$x_{vm}^{(n+1)} = x_{vm}^{(n)} \frac{1}{\sum_{d=1}^{N_d} p_{vd} - \lambda_{vm}} \sum_{d=1}^{N_d} \frac{p_{vd} s_{dm}}{\sum_{v'=1}^{N_v} p_{v'd} x_{v't}^{(n)}}$$

$$\lambda_{vm} = \frac{F(\xi_k^{(n)})_m - x_{vm}^{(n)}}{\sigma^2}$$

STEP 2

1) **CUDA kernels** to *evaluate kinetic model and derivatives* of log posterior and of *MRF log prior*.
2) **CUDA** implementation of the **MAP-LM** optimization, based on **cuBLAS** batched matrix operations.

$$\xi^{n+1} = \xi^n + \{(J^T J + \lambda I)^{-1} [J^T (x - F(\xi)) - \beta g_{prior}]\}$$

The entire EM-LM reconstruction is performed on the GPU, with one GPU thread per voxel.

Insight about GPU implementation of MAP-LM optimization algorithm

iterate until convergence

STEP 1

Many GPU threads working in parallel on a CUDA kernel to **evaluate model and derivatives**, voxelwise [2].

$$F(\xi, t) = \sum_{c=1}^{2} \frac{A_1 t \alpha_c}{\beta_c - \lambda_1} e^{-\lambda_1 t} + \sum_{c=1}^{2} \sum_{j=1}^{3} \frac{\hat{A}_j t \alpha_c}{\beta_c - \lambda_j} \left(e^{-\lambda_j t} - e^{-\beta_c t}\right)$$

STEP 2

cuBLAS functions, implementing a parallel *Levenberg-Marquardt* optimization, through **batched matrix multiplications and inversions**.

cuBLAS

STEP 3

Many GPU threads working in parallel on a CUDA kernel to compute the **derivatives of the Markov Random Field log prior**, with **Smooth L1-norm cost function** on the current estimate of kinetic maps.

STEP 4

Parameter estimates update, voxelwise (*kinetic maps*).

$$\xi^{n+1} = \xi^n + \{(J^T J + \lambda I)^{-1} [J^T (x - F(\xi)) - \beta g_{prior}]\}$$

	Speed $(TACs/s)$	Time $Volume$ [128×128×128]
Comkat-like	$\sim 0,7$	$\sim 35 days$
Analytic	~ 80	$\sim 7.5 hours$
Parallel CPU	~ 900	$\sim 40 min$
Parallel GPU (CUDA)	$\sim 300,000$	$\sim 7 sec$

Simulations

Simulation setup

To assess the effect of the KCS algorithm in comparison with standard kinetic modeling techniques, and to evaluate the performance of the GPU implementation, we realized a ***Monte Carlo (MC) simulation with 100 noisy realizations*** of a simple geometrical phantom. The kinetic behavior of the three main regions has been simulated using a ***2-tissues irreversible compartment model*** *(i.e. 3 parameters model)*, while the square area in the center has been modeled as a blood input region. In this simulation study we generated **synthetic dynamic PET data**, according to the hierarchical bayesian model presented above.

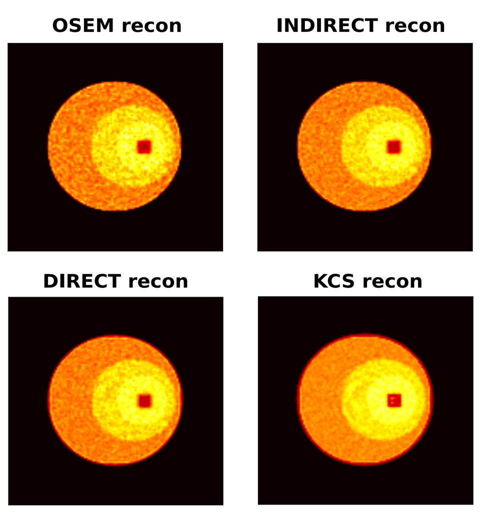


Fig. 2

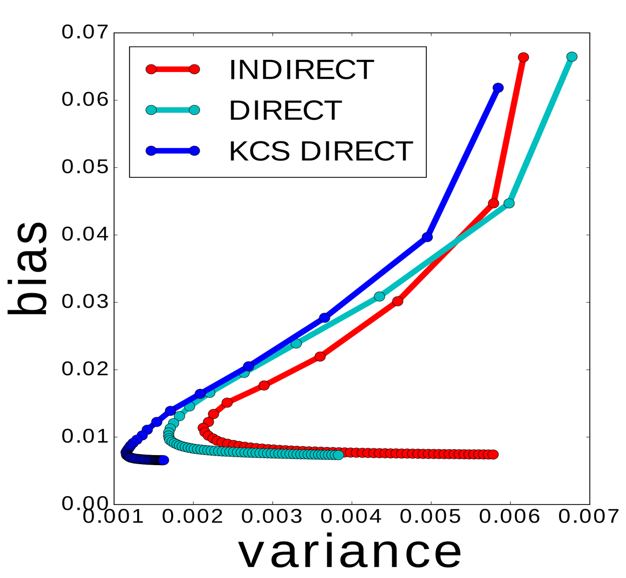


Fig.3

Results

We compared the results of **three different methods** (*indirect recon, direct recon, and direct recon with kinetic compressive sensing, KCS*).

In **Fig.2**, it is easy to recognize a first ***reduction in voxel-by-voxel variance*** when the kinetic model is used to regularize the reconstruction (***DIRECT***), which is further reduced when the sparsity assumption of the spatial derivatives of the parametric maps is enforced (***KCS***).

The **bias/variance** plot shows how a direct approach improves the quality of the estimate of parametric maps, with respect to the results provided by a standard indirect post-reconstruction fitting, but also how the novel sparsity constraint is able to **further reduce the variance** of the produced parametric maps, **without affecting** (if not decreasing) **the bias**.

Human Data

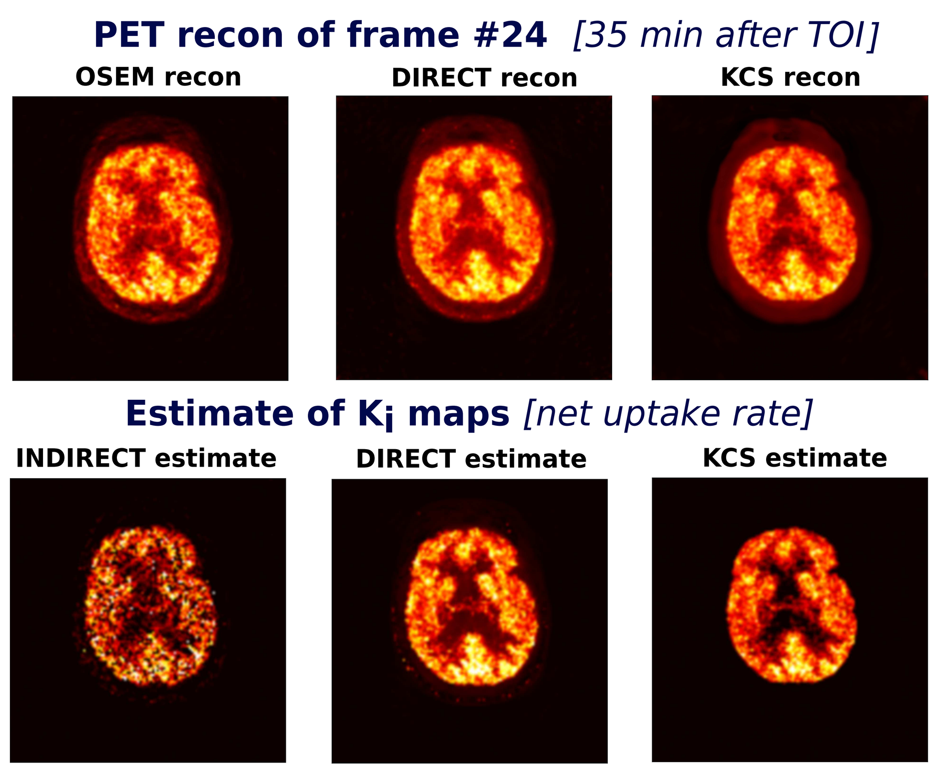


Fig.4

PET Dataset

The conventional indirect and direct, and the novel KCS approaches were applied to **(18F)-FDG brain PET data**, acquired on a **Siemens mMR PET-MR scanner**, using a *2-tissue irreversible compartment model.*

PET Results

Top row of **Fig.4** shows how the different methods perform in terms of **image reconstruction**, while the ***bottom row*** show the estimated **K_i (net uptake rate)** parametric maps: the proposed KCS direct method is able to produce spatially coherent images, with **low noise** and **good tissue contrast**, also when it comes to parametric maps estimates.

Conclusions

The simulation study demonstrated that the proposed method of **introducing a sparsity-inducing prior in a direct reconstruction framework** can help in producing high-quality images and parametric maps, which are both amenable for display and quantitatively more accurate than what a post-reconstruction fitting and unconstrained direct reconstruction can achieve (i.e. lower bias and lower variance, Fig.3).
This method appears to be promising as a **feasible approach for applying kinetic modeling to very large 4D clinical datasets** with a ***reduced computational cost***, thanks to the parallel GPU implementation based on the analytic expression of the kinetic model and its derivatives.
Future studies will extend the current open-source implementation, by integrating different kinetic models (linear and non-linear) and different priors.
The proposed approach can also be adapted not only to PET data, but also to different dynamic imaging techniques, such as **dynamic CT** or **dynamic contrast enhancement (*DCE*)** MRI.

References

(1) A.J. Reader, and J. Verhaeghe. *4D image reconstruction for emissiontomography*. Physics in medicine and biology, 2014.
(2) B.R. Smith, G. Hamarneh and A. Saad. *Fast GPU Fitting of KineticModels for Dynamic PET*. MICCAI High Performance Workshop, 2010.
(3) J. Besag. *On the statistical analysis of dirty pictures*. J.Roy.Stat., 1986.
(4) P.J. Green. *On use of the EM algorithm for likelihood estimation*. J. Roy. Stat., 1990.
(5) S. Pedemonte, C. Catana, and K. Van Leemput. *An Inference Languagefor Imaging.* BAMBI. Springer International Publishing, 2014.
(6) P.J. Huber. *Robust Statistics*. John Wiley & Sons. New York, NY, USA: 1981.

Michele Scipioni
PhD student, University of Pisa
GRA, MGH/Martinos Center, Boston

email: michele.scipioni@ing.unipi.it
tel: +39 320 247 5453