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The influence of noise in dynamic PET direct reconstruction

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Aim

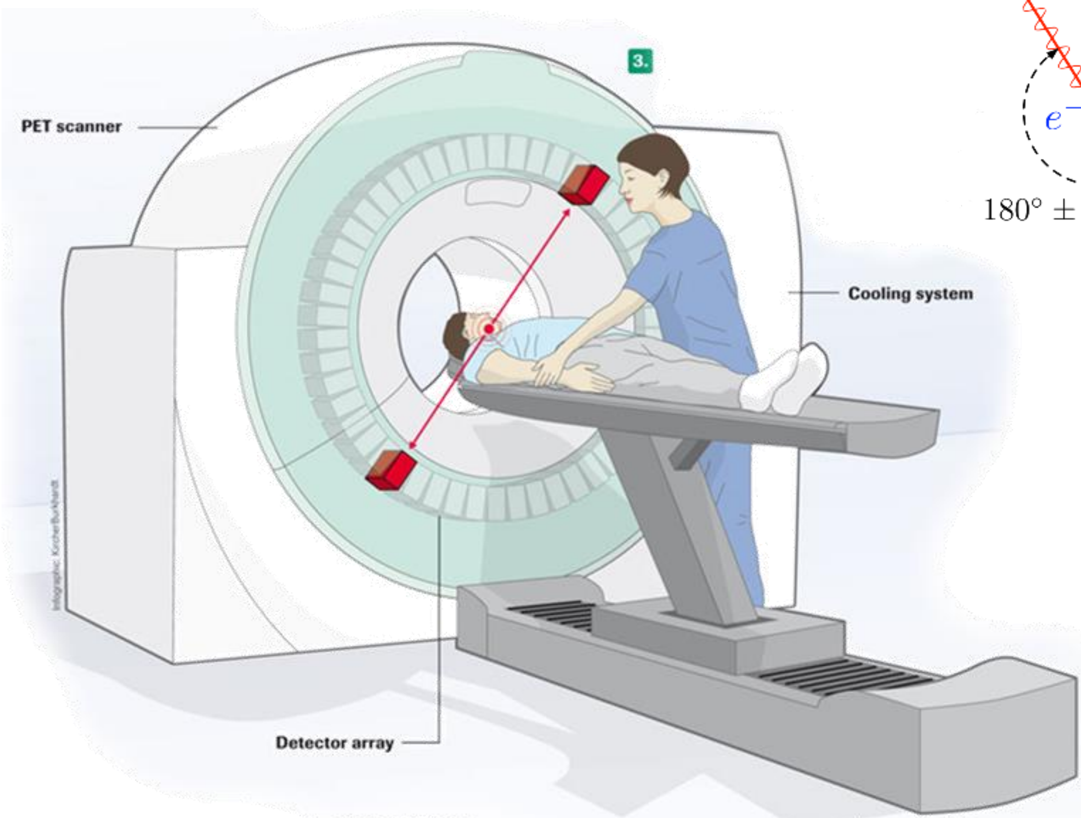
Direct reconstruction methods are one of the most up-to-date topic in PET research and several different algorithms have been presented in the last few years.

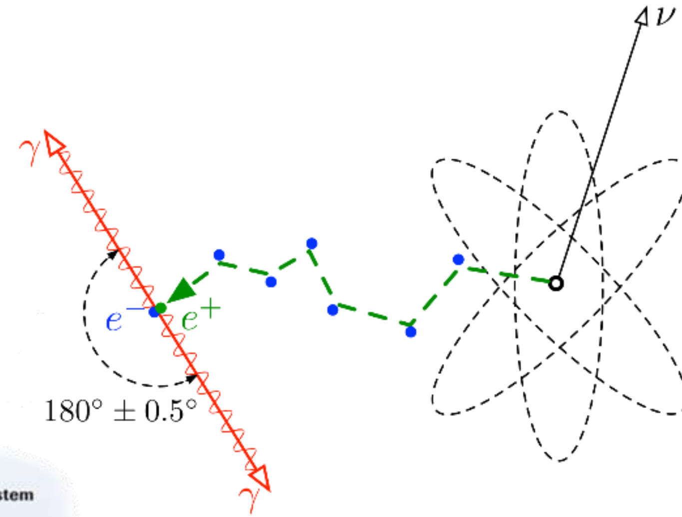
However, **no studies have been performed so far about the evaluation of the performance of this new class of direct reconstruction algorithms when noisy data are considered**. In fact, it is well known that the presence of noise sources compromises the estimation of the emission density when ML reconstruction algorithms are used.

In the present work we study the behavior of a particular direct reconstruction algorithm, starting from dynamic PET data with different noise degrees. Such evaluation is performed by **simulating realistic PET measured data, adding the effects of different noise sources** and analyzing them with new approach.

Background

Positron Emission Tomography (PET)

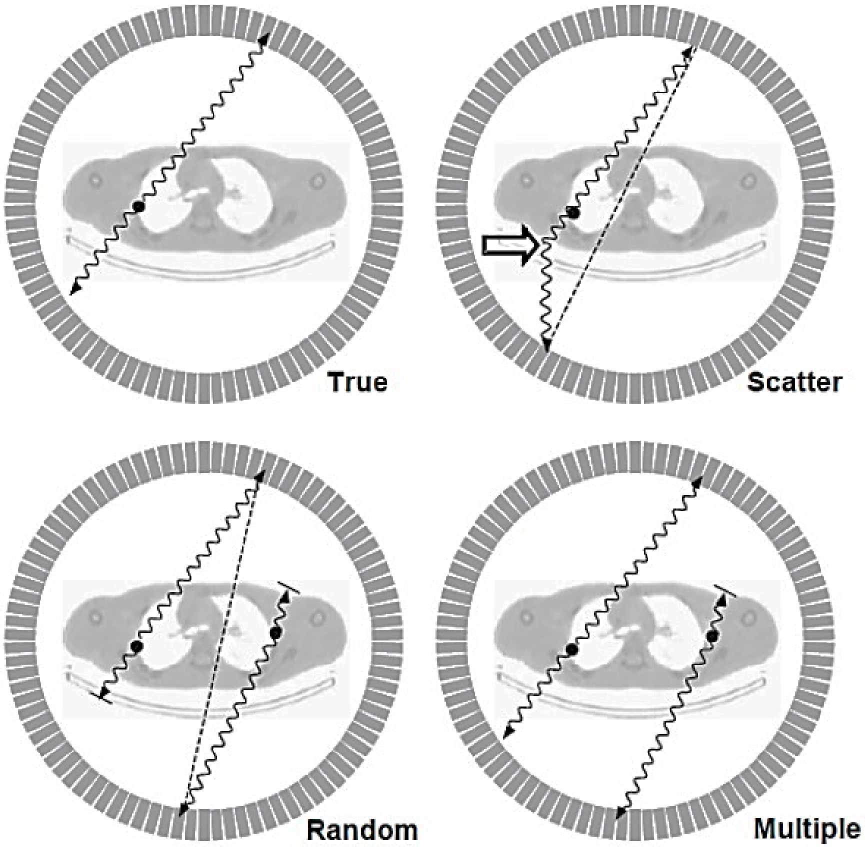




- e^+ positron
- e^- electron
- ν neutrino
- γ quantum/photon (511 keV)

Noise sources

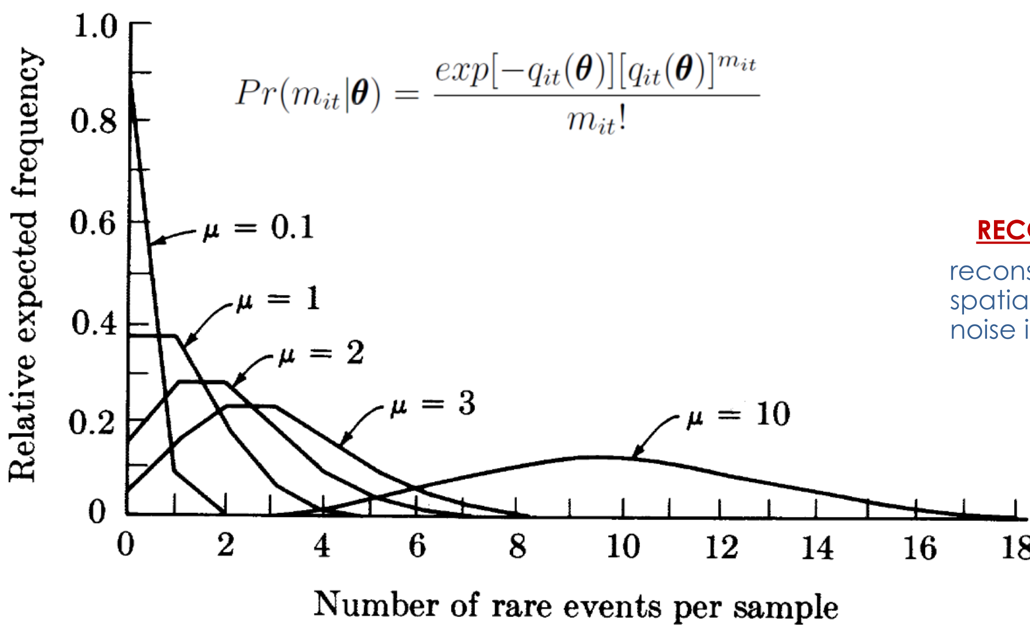
Noise can be categorized as **structured** or **unstructured** noise.



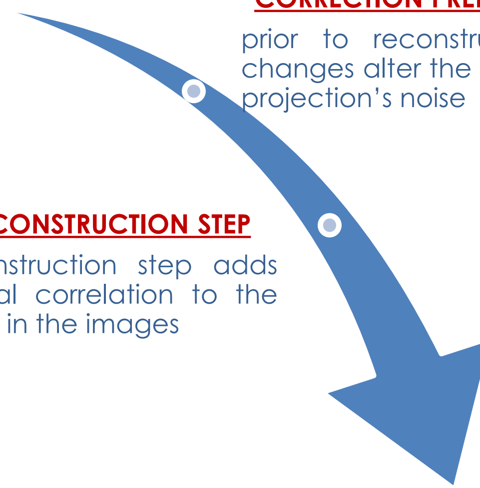
Random statistical variations in the counting rate (**Poisson counting noise**), modulated by applied correction and the chosen reconstruction algorithm.

Statistical distribution of noise

The *random process of photon detection* generates a variation in the counts that can be described with a Poisson distribution. This is actually the main cause of noise!



$$Pr(m_{it}|\boldsymbol{\theta}) = \frac{exp[-q_{it}(\boldsymbol{\theta})][q_{it}(\boldsymbol{\theta})]^{m_{it}}}{m_{it}!}$$



UNCORRECTED RAW DATA

nonstationary noise but uncorrelated and with a known Poisson nature

CORRECTION PREPROCESSING

prior to reconstruction these changes alter the distribution of projection's noise

RECONSTRUCTION STEP

reconstruction step adds spatial correlation to the noise in the images

PET IMAGES

The noise is characterized by un unknown statistical distribution and all we can do is to make assumptions to model it

Dynamic functional imaging

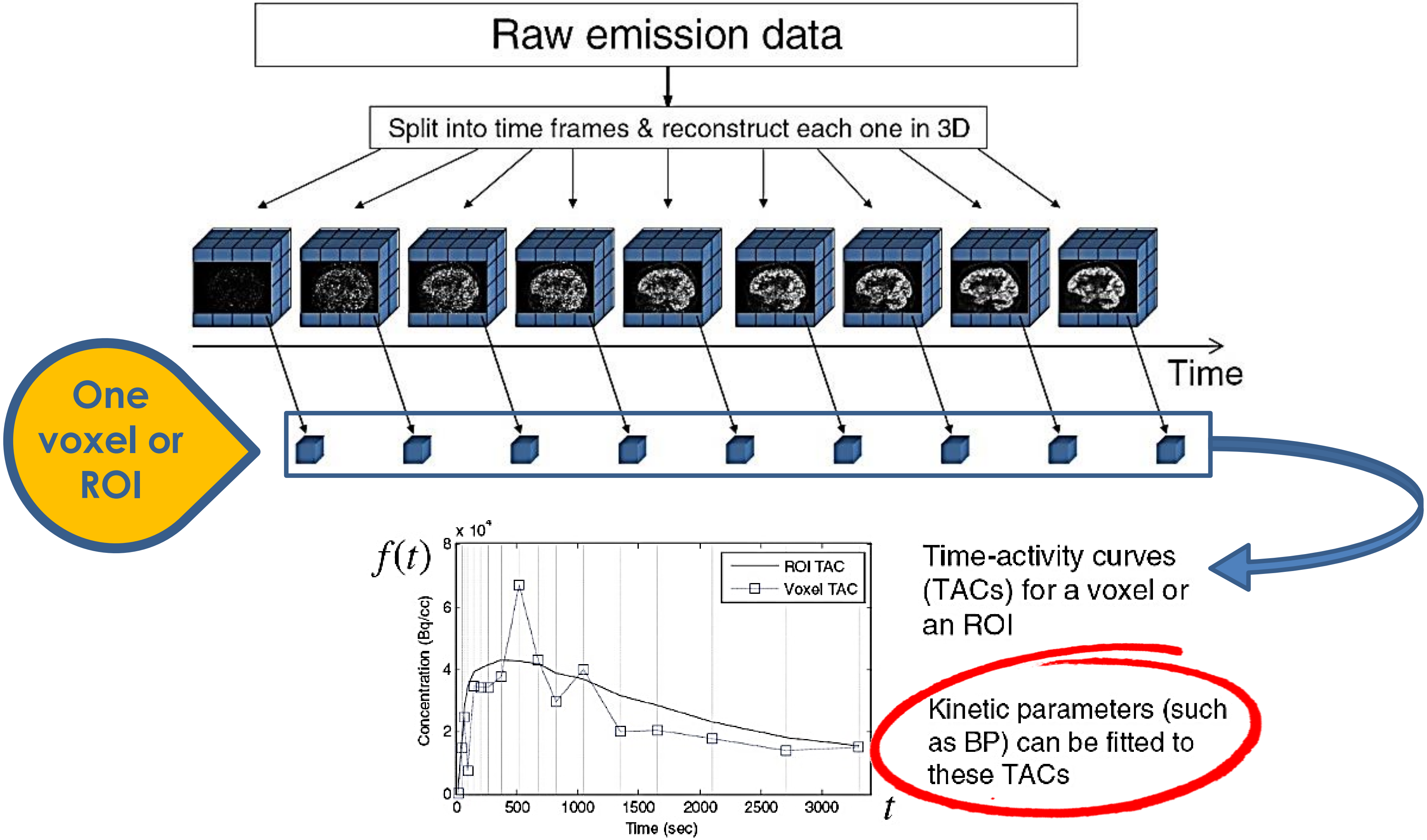
Why?

Dynamic studies are performed to **quantify tissue-specific biochemical properties**. When acquiring a dynamic PET scan, the activity of the PET tracer is measured at multiple time points, involving a sequence of acquisitions.

*Monitor the tracer's **distribution in and metabolization** by the tissues.*

*Estimate important **metabolic parameters** (blood flow, binding potentials, metabolic rates, …)*

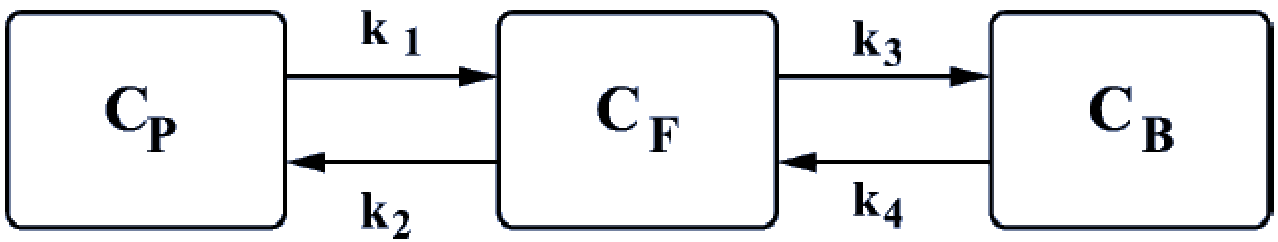
Conventional analysis of dynamic sequences



Time-activity curves (TACs) for a voxel or an ROI

Kinetic parameters (such as BP) can be fitted to these TACs

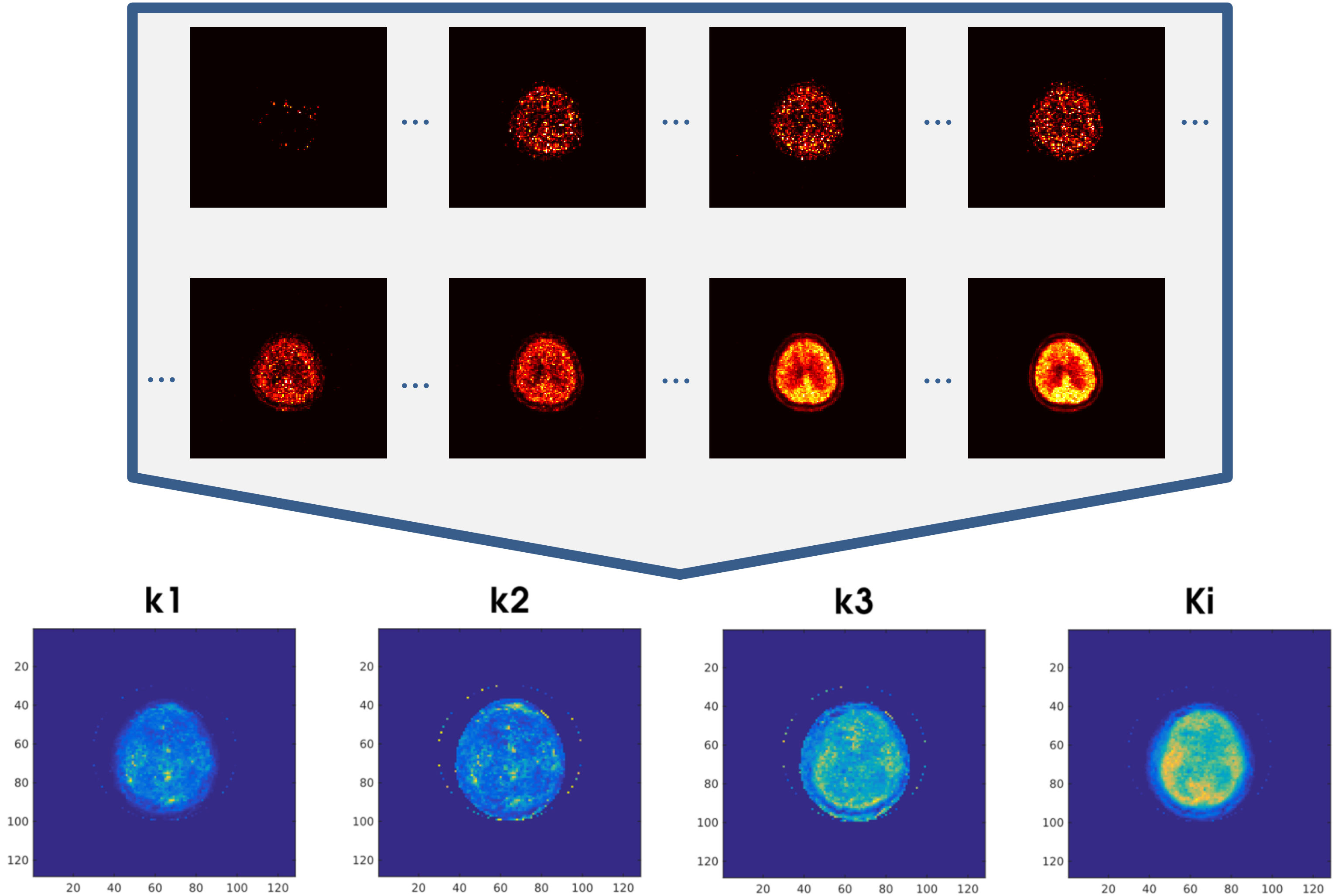
Compartmental model



$$TAC(t, k, f_v) = (1 - f_v)h(t, k) \otimes C_p(t) + f_v C_{wb}(t)$$

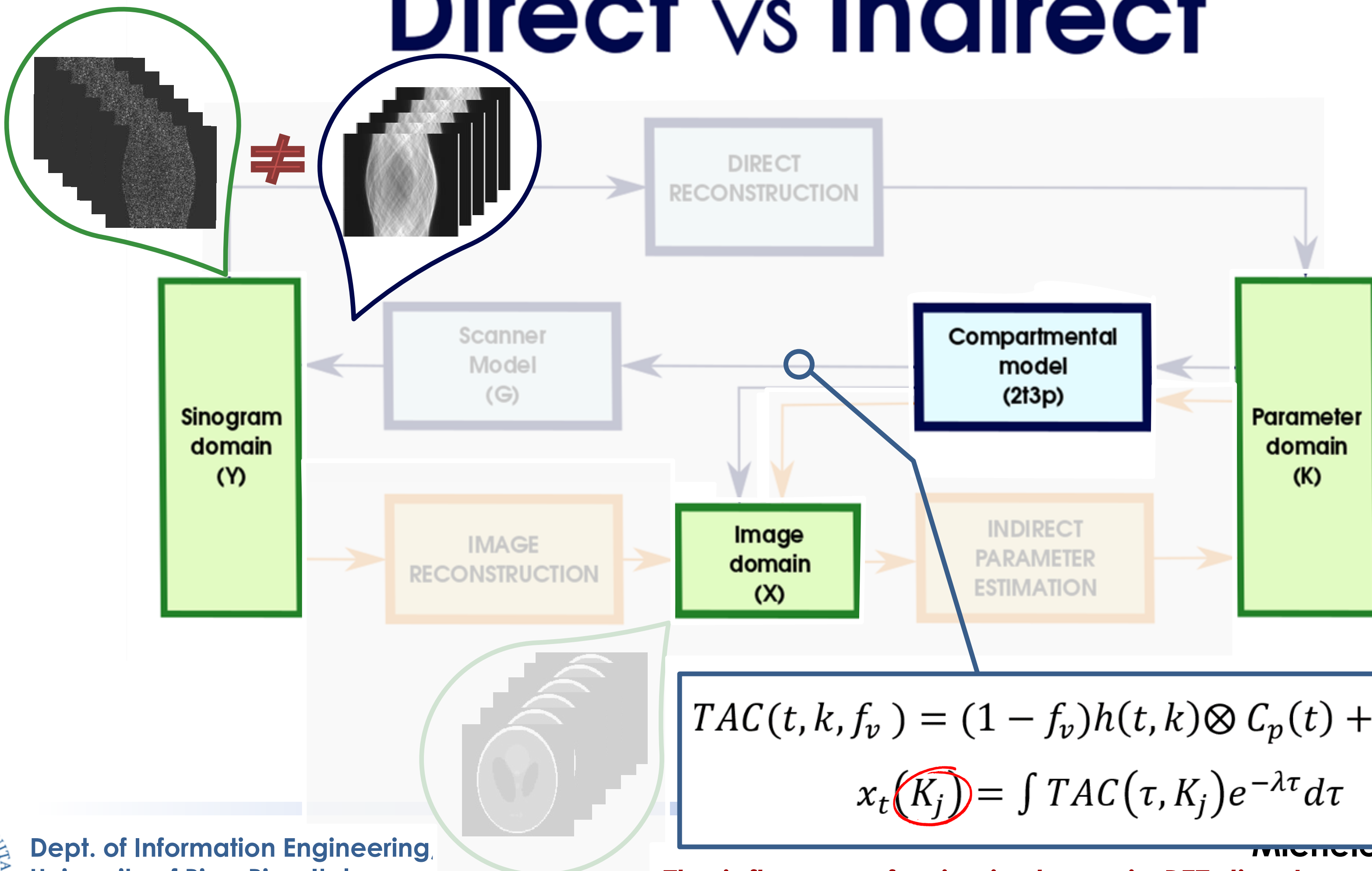
Kinetic analysis in a voxel-by-voxel fashion provides parametric images that can be used to determine the spatial distribution and metabolitation of the specific tracer.

Compartmental model



Direct parametric images estimation

Direct vs Indirect



$$TAC(t,k,f_v) = (1-f_v)h(t,k) \otimes C_p(t) + f_v C_{wb}(t)$$

$$x_t(K_j) = \int TAC(\tau, K_j) e^{-\lambda\tau} d\tau$$

Optimization transfer

In order to estimate the updated parameter matrix K, we have to **evaluate** the following **log-likelihood function**:

$$\log \lambda^{(MAP)}(\boldsymbol{\theta}|\boldsymbol{m}) = \sum_{t=1}^{T}\sum_{i=1}^{I}\left(m_{it}\log q_{it}(\boldsymbol{\theta}) - q_{it}(\boldsymbol{\theta})\right) - \beta U(\boldsymbol{\theta}) + const$$

measured sino (m_{it}) — *expected sino* ($q_{it}(\boldsymbol{\theta})$) — *penalization term* ($\beta U(\boldsymbol{\theta})$)

The proposed method finds the solution via an **optimization transfer approach** and dividing the update in 2 different steps:

$$\boldsymbol{f}_{EM}^{k} = \frac{\boldsymbol{f}(\boldsymbol{\theta}^k)}{\boldsymbol{A}^T\mathbf{1}}\boldsymbol{A}^T\frac{\boldsymbol{m}}{\boldsymbol{A}\boldsymbol{f}(\boldsymbol{\theta}^k)+\boldsymbol{\rho}}$$

Frame-wise EM-like image update

$$\boldsymbol{\theta}^{k+1} = \underset{\boldsymbol{\theta}}{\mathrm{argmax}}\sum_{vt}\left[s\left(\boldsymbol{f}_{EM}^{k}\ln \boldsymbol{f}(\boldsymbol{\theta}) - \boldsymbol{f}(\boldsymbol{\theta})\right)\right]_{vt} - \beta U(\boldsymbol{\theta})$$

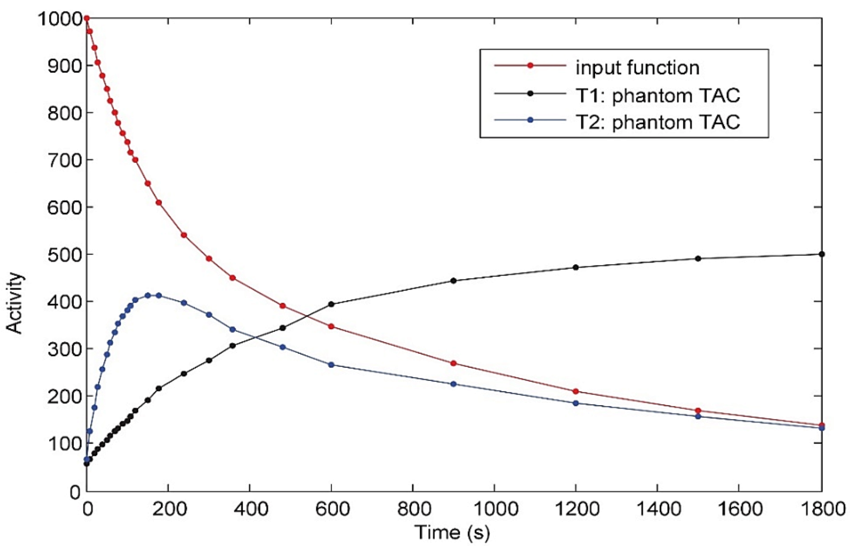
Voxel-wise penalized likelihood fitting

Simulation

Simulated Dataset

Raw data are generated by projection of 2D radioactivity distribution functions into sinograms (true coincidences), adding random and scatter coincidences, and measurement noise. For each emission time frame, Poisson events are generated.

	K1	k2	k3	k4	Ki	fv
T1	0,082	0,055	0,085	0,002	0,0497	0,05
T2	0,426	0,660	0,010	0,022	0,0064	0,03



Radius	10 cm
Length	15 cm
FOV	70 cm
Image dimension	128x128 px
Sino dimension	186x360 px

MonteCarlo simulations

We performed 50 repetitions of a Monte Carlo simulations changing the level of noise added to the simulated data, for each one of the main sources.

- ***Accidental Scattering (AC)*** *were generated as Poisson events identically distributed in the sinogram, with a constant mean value;*
- ***Random counts (RS)*** *in the sinogram was modelled as a Gaussian function having its maximum at the center of each projection, and extending to the tails, which are outside the source boundary;*
- ***Gaussian measurement noise*** *values (GN) are the means of a Poisson events generator*

	Min	Max
AC	0%	30%
RS	0%	30%
GN	2%	

Mean value of each noise source as a % of max sinogram's value

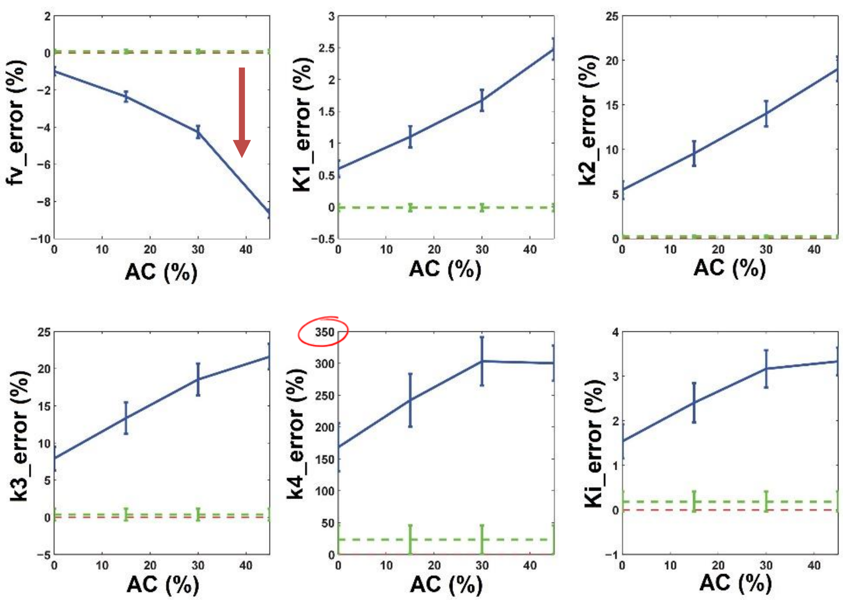
Results

K1	k2	k3	k4
0,082	0,055	0,085	0,002

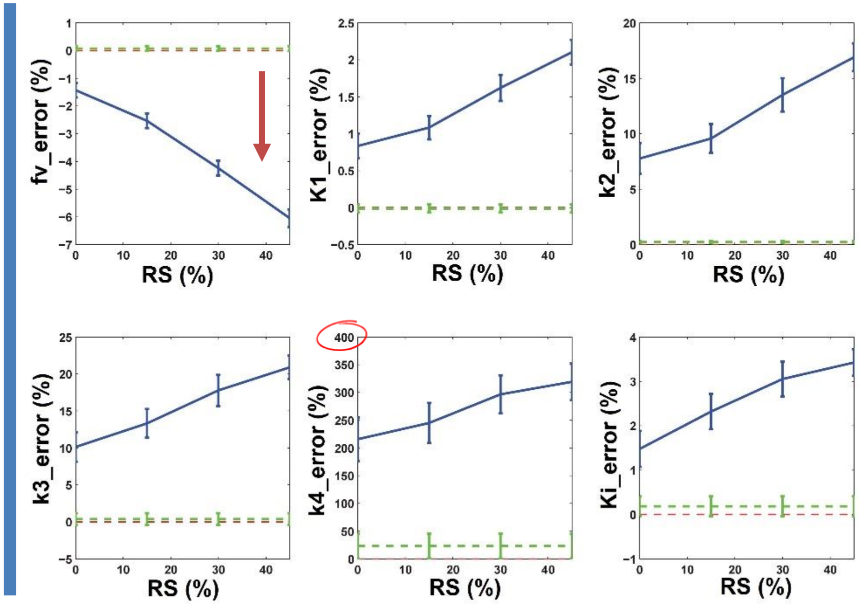
Simulation 1 (T1)

Effect of accidental scattering and random counts on kinetic parameters estimation for both simulated phantoms.

$$\%k_j = \frac{\hat{k}_j - k_j}{k_j}, j = 1 \dots 4 \qquad \%f_v = \frac{\hat{f}_v - f_v}{f_v}$$



Accidental Scattering



Random Counts

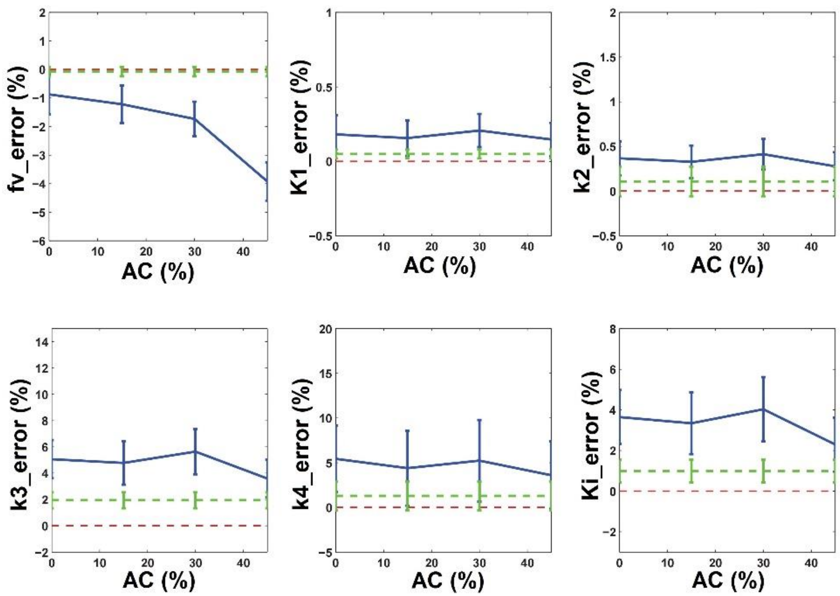
K1	k2	k3	k4
0,426	0,660	0,010	0,022

Simulation 2 (T2)

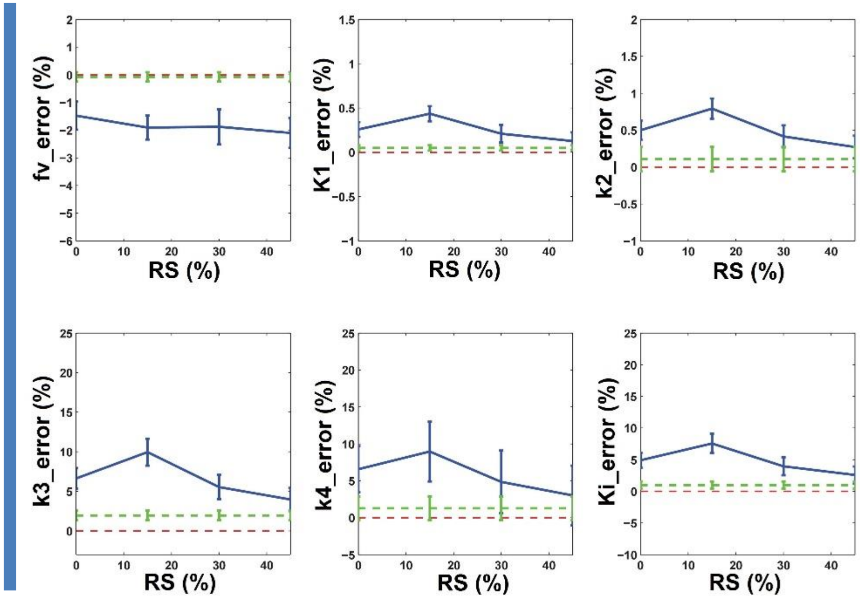
Effect of accidental scattering and random counts on kinetic parameters estimation for both simulated phantoms.

$$\%k_j = \frac{\hat{k}_j - k_j}{k_j}, j = 1 \dots 4$$

	K1	k2	k3	k4
T1	0,082	0,055	0,085	0,002
T2	0,426	0,660	0,010	0,022



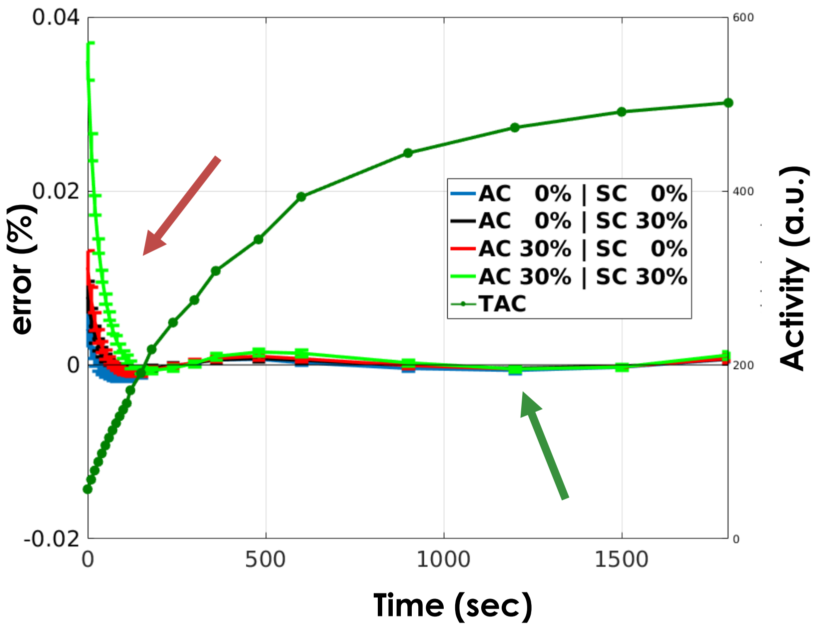
Accidental Scattering



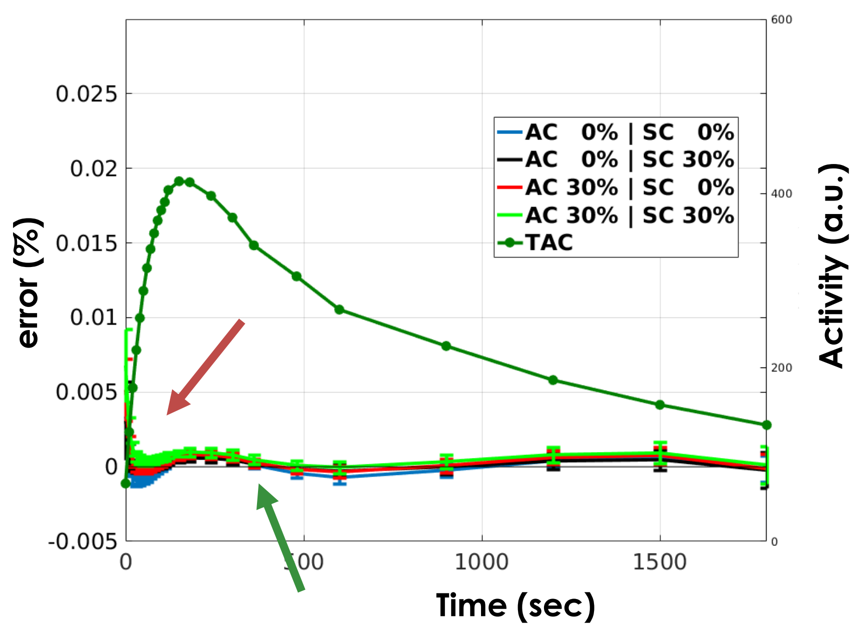
Random Counts

Dynamic images error

$$E\left(\frac{\hat{X}(t)-X(t)}{X(t)}\right), t=1\ldots T_{tot}$$



Simulated tissue 1



Simulated tissue 2

Conclusions

Summary and future development

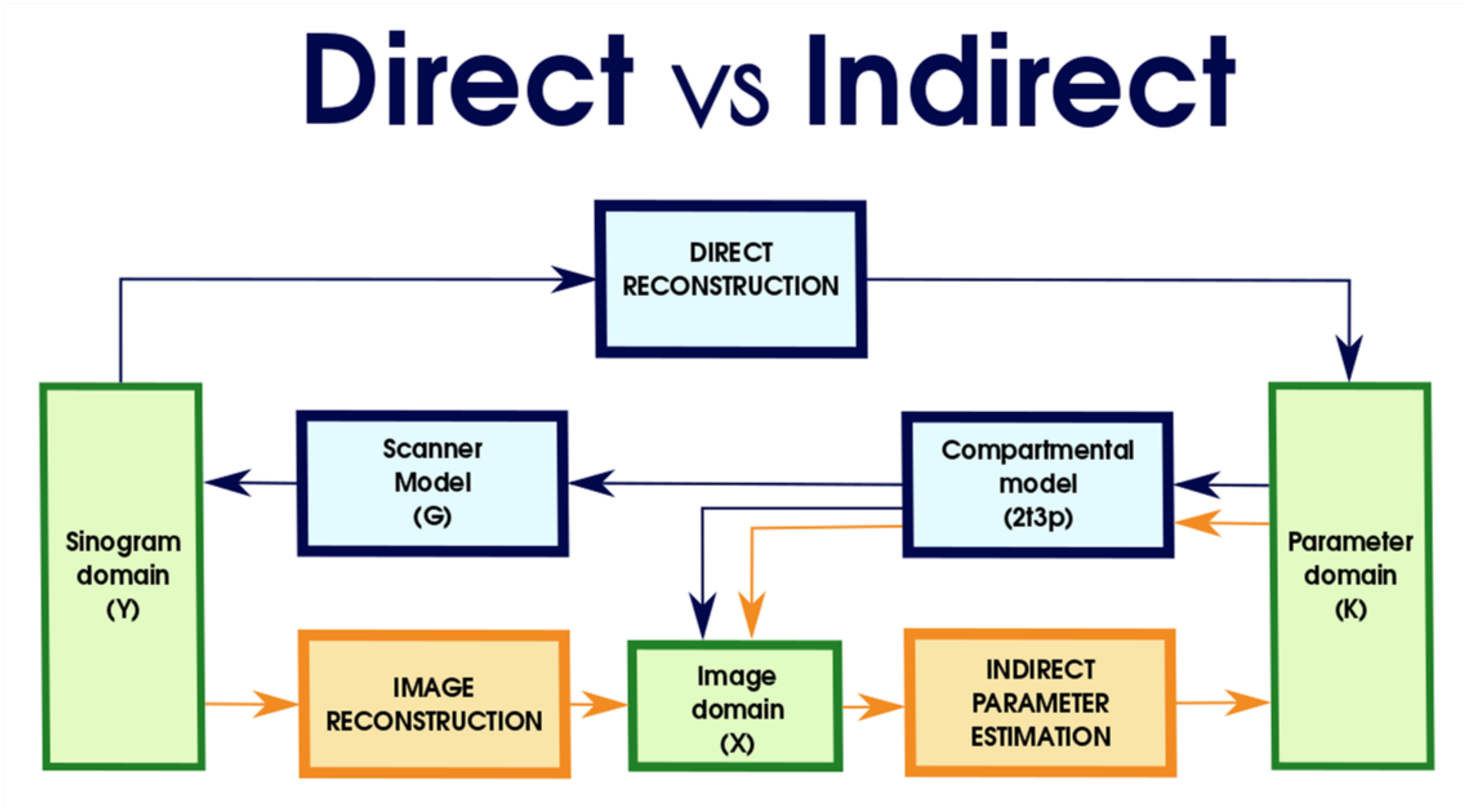
In this work *the behavior of a dynamic PET direct reconstruction algorithm on noisy data has been studied*. We performed simulations in order **to extract indexes that quantitatively describe the goodness** of *kinetic parameters estimation* and *dynamic images reconstruction*.

The direct reconstruction algorithm tested grant **good performance** *also in presence of noise* on simulated data due to both accidental scattering and random counts.

THE OBJECTIVE OF THE FUTURE WORK WILL BE TO DEFINE A NEW METHOD FOR ASSESSING THE ERROR IN PARAMETER ESTIMATION FROM THE KNOWLEDGE OF THE NOISE THAT AFFECTS MEASURED DATA.

Thank you
for your attention!

Appendix – Direct vs Indirect



Appendix - Optimization transfer

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measured sino — *expected sino* — *penalization term*

The proposed method finds the solution via an **optimization transfer approach** and dividing the update in 2 different steps:

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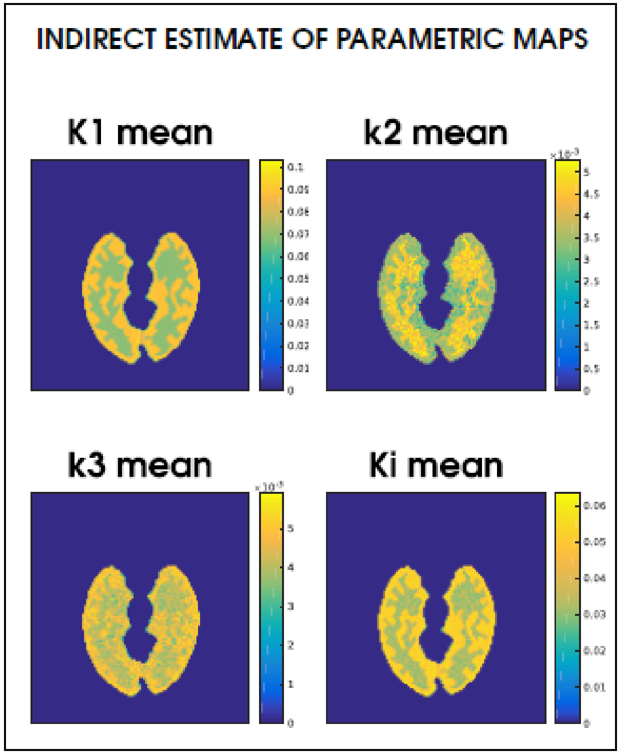
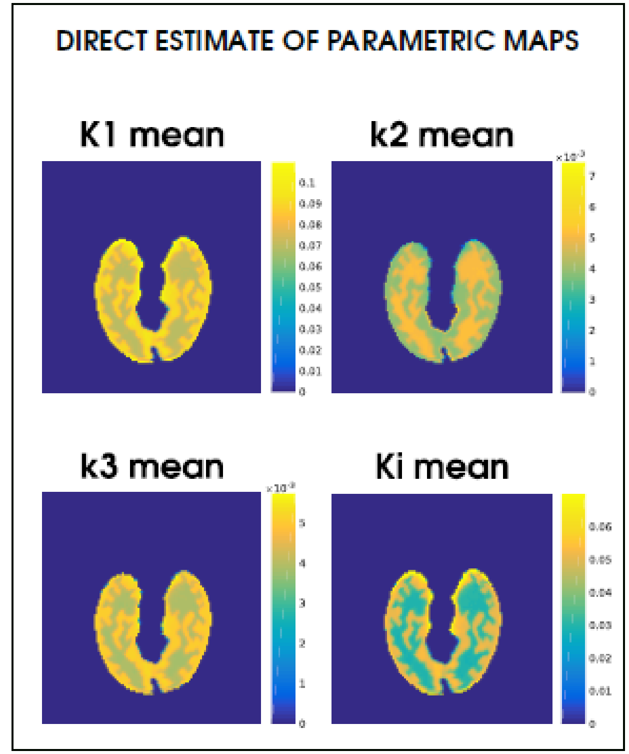
Frame-wise EM-like image update

$$\boldsymbol{\theta}^{k+1} = \underset{\boldsymbol{\theta}}{\operatorname{argmax}}\sum_{vt}\left[s\left(\boldsymbol{f}_{EM}^{k}\ln \boldsymbol{f}(\boldsymbol{\theta}) - \boldsymbol{f}(\boldsymbol{\theta})\right)\right]_{vt} - \beta U(\boldsymbol{\theta})$$

Voxel-wise penalized likelihood fitting

Appendix – EMIM16, Utercht 7-10 March 2016





NORMALIZED ROOT MEAN SQUARE (nRMSE)

$$\text{normalized RMSE}(k_i) = \frac{\sqrt{\frac{1}{|S|}\sum_{s\in S}\left(k_{i,s}^{\text{true}} - k_{i,s}^{\text{DIRECT}}\right)^2}}{\sqrt{\frac{1}{|S|}\sum_{s\in S}\left(k_{i,s}^{\text{true}} - k_{i,s}^{\text{INDIRECT}}\right)^2}}$$

K1 = 0.7722 ± 0.0024
k2 = 0.7847 ± 0.0077
k3 = 1.0003 ± 5.16e-05
KI = 0.9960 ± 1.3e-04