Kinetic compressive sensing:
improving image reconstruction and parametric maps

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Background

Parametric images provide insight into the spatial distribution of physiological parameters, but they are often extremely noisy, due to *low SNR of tomographic data*. **Direct estimation from projections** [1] allows accurate noise modeling, improving the results of post-reconstruction fitting. We propose a method, which we name **kinetic compressive sensing (KCS)**, based on a *hierarchical Bayesian model* and on a novel reconstruction algorithm, that encodes **sparsity of kinetic parameters**.

Hierarchical Bayesian Model

The model has three key components:

1) the **model of the acquisition system** consists of the ordinary Poisson model, incorporating all effects of attenuation, scatter and randoms;
2) the **kinetic model** encodes the assumption that the voxel intensities are noisy realizations of a *hidden dynamic process*, modeled using a *multi-compartmental model;*
3) a **sparsity-inducing prior** distribution of the kinetic parameters is introduced as a *Markov Random Field (MRF) with Smooth L1-norm cost function* [6].

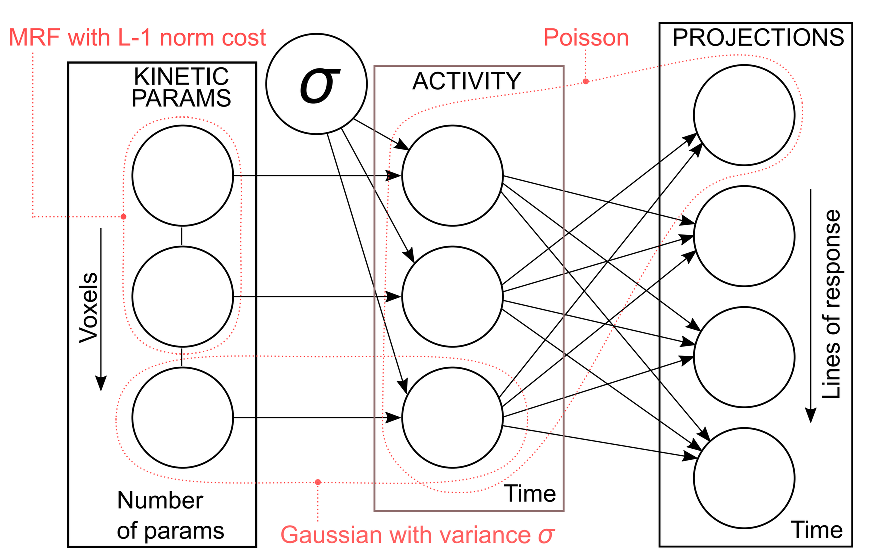
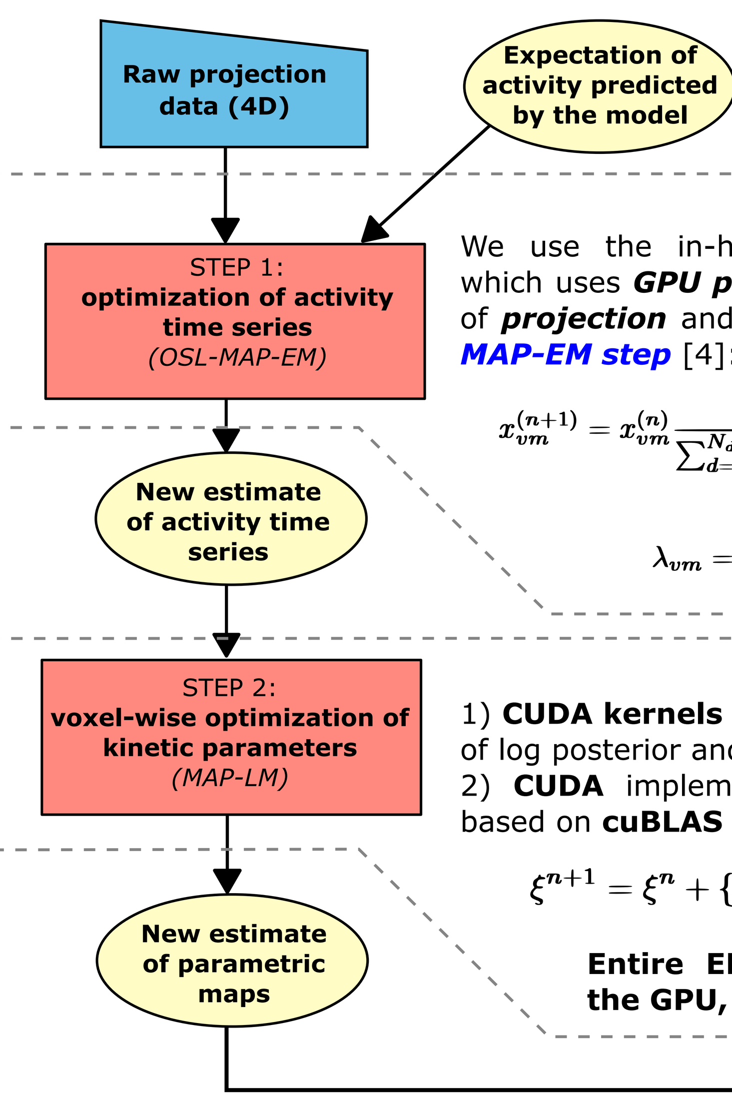


Fig. 1

Algorithm workflow (ICM [3])



STEP 1

We use the in-house developed ***Occiput***[5] **toolbox**, which uses ***GPU parallel computation*** for the operations of ***projection*** and ***backprojection***, involved in the ***OSL-MAP-EM step*** [4]:

$$x_{vm}^{(n+1)} = x_{vm}^{(n)} \frac{1}{\sum_{d=1}^{N_d} p_{vd} - \lambda_{vm}} \sum_{d=1}^{N_d} \frac{p_{vd} s_{dm}}{\sum_{v'=1}^{N_v} p_{v'd} x_{v't}^{(n)}}$$

$$\lambda_{vm} = \frac{F(\xi_k^{(n)})_m - x_{vm}^{(n)}}{\sigma^2}$$

STEP 2 (iterative)

1) **CUDA kernels** to evaluate kinetic model and derivatives of log posterior and of MRF log prior.
2) **CUDA** implementation of the ***MAP-LM*** optimization, based on **cuBLAS** batched matrix operations.

$$\xi^{n+1} = \xi^n + \{(J^T J + \lambda I)^{-1} [J^T (x - F(\xi)) - \beta g_{prior}]\}$$

Entire EM-LM reconstruction performed on the GPU, with one GPU thread per voxel.

Insight about GPU implementation of MAP-LM optimization algorithm

iterate until convergence

STEP 1

M GPU threads working in parallel on a CUDA kernel to **evaluate model and derivatives**, voxelwise [2].

$$F(\xi, t) = \sum_{c=1}^{2} \frac{A_1 t \alpha_c}{\beta_c - \lambda_1} e^{-\lambda_1 t} + \sum_{c=1}^{2} \sum_{j=1}^{3} \frac{\hat{A}_j t \alpha_c}{\beta_c - \lambda_j} \left(e^{-\lambda_j t} - e^{-\beta_c t}\right)$$

STEP 2

cuBLAS functions implementing a parallel *Levenberg-Marquardt* optimization, through **batched matrix multiplications and inversions.**

cuBLAS

STEP 3

M GPU threads working in parallel on a CUDA kernel to compute the **derivatives of the Markov Random Field log prior**, with **Smooth L1-norm cost function** on the current estimate of kinetic maps.

STEP 4

Parameter estimate update, voxelwise.

$$\xi^{n+1} = \xi^n + \{(J^T J + \lambda I)^{-1} [J^T (x - F(\xi)) - \beta g_{prior}]\}$$

	Speed $(TACs/s)$	Time Volume [128×128×128]
Comkat-like	~ 0,7	~ 35*days*
Analytic	~ 80	~ 7.5*hours*
Parallel CPU	~ 900	~ 40*min*
Parallel GPU (CUDA)	~ 300,000	~ 7*sec*

Simulations

Simulation setup

To assess the effect of the KCS algorithm in comparison with standard kinetic modeling techniques, and to evaluate the performance of the GPU implementation, we realized a ***Monte Carlo (MC) simulation with 100 noise realizations***. The kinetic behavior of the three main regions has been simulated using a ***2-tissues irreversible compartment model***, while the square area in the center has been modeled as a blood input region. In this simulation study we generated **synthetic dynamic PET data**, according to the hierarchical bayesian model presented.

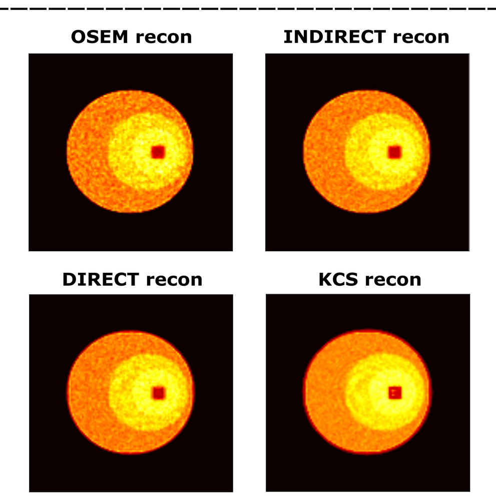


Fig. 2

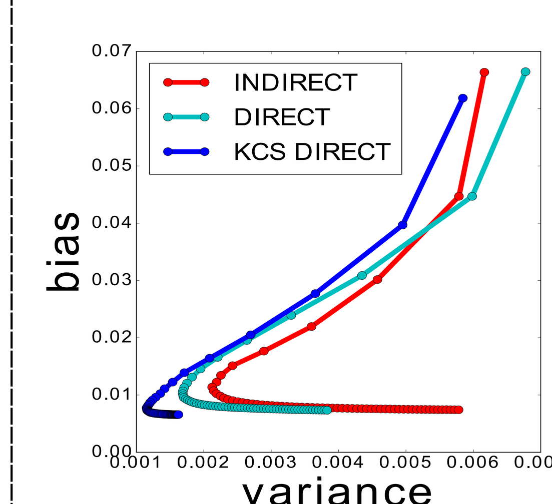


Fig.3

Results

We compared the results of **three different methods** (*indirect recon, direct recon, and direct recon with kinetic compressive sensing, KCS*). In Fig.2, it is easy to recognize a first reduction in voxel-by-voxel variance when the kinetic model is used to regularize the reconstruction (***DIRECT***), which increases when the sparsity assumption of the spatial derivatives of the parameters is enforced (***KCS***). The **bias/variance plot** shows how a direct approach improves the quality of the estimate of parametric maps, with respect to the results provided by a standard indirect post-reconstruction fitting, but also how the novel sparsity constraint is able to **further reduce the variance** of the produced parametric maps, **without affecting the bias**.

Human Data

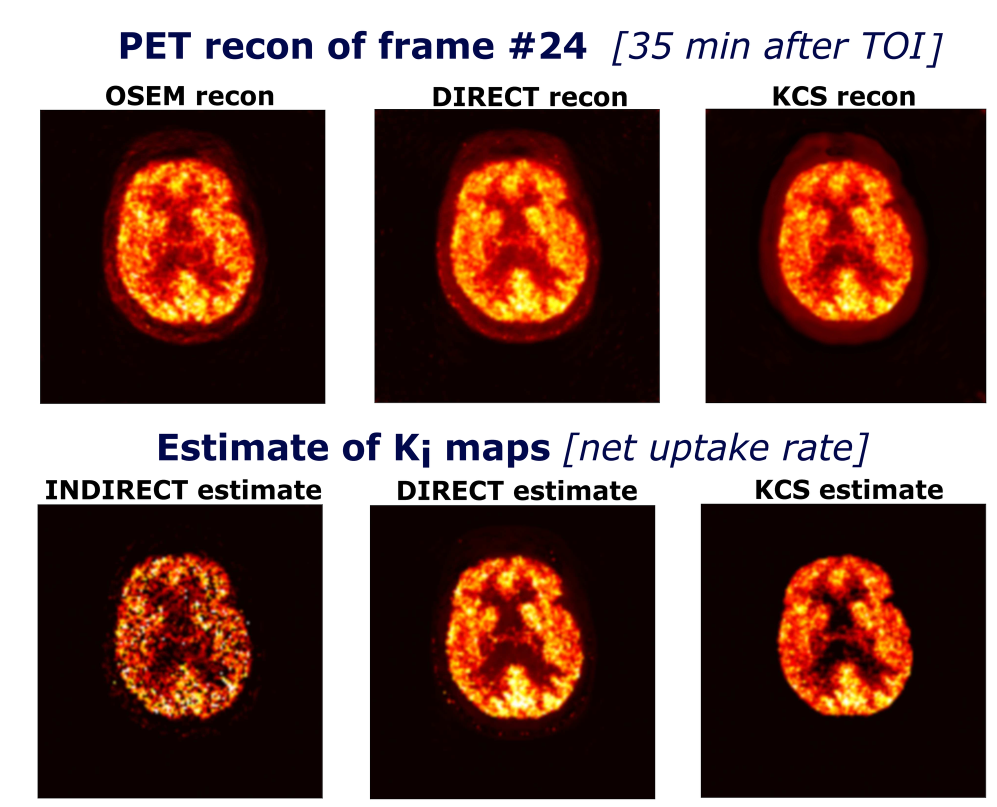


Fig.4

PET Dataset

The conventional indirect and direct, and the novel KCS approaches were applied to **[18F]-FDG brain PET data**, acquired on a **Siemens mMR PET-MR scanner**, using a *2-tissue irreversible compartment model.*

PET Results

Top row of FIG.4 shows how the different methods perform in terms of **image reconstruction**, while the ***bottom row*** show the estimated **K_i (net uptake rate)** parametric maps: the proposed KCS direct method is able to produce spatially coherent images, with **low noise** and **good tissue contrast**, also when it comes to parametric maps estimates.

[*Preliminary*] DCE-MRI study

We applied a modified version of the KCS method to a dataset of **oncologic liver Gd-DTPA DCE-MRI**. With respect to Fig.1, now the *relationship between k-space and image domain* is governed by the **NUFFT transform**, while the kinetic model used is a **1-tissue compartment model.** Comparing the reconstructions (Fig.5) of an early and short time frame (***highly subsampled k-space***), it is possible to appreciate how the KCS helps ***pointing out lesions*** *that, with a standard reconstruction, are completely lost*.

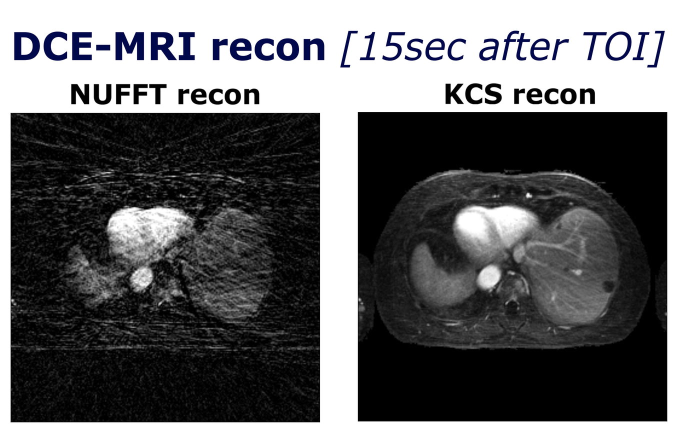


Fig.5

Conclusions

The simulation study demonstrated that the proposed method of **introducing a sparsity-inducing prior in a direct reconstruction framework** can help in producing high-quality images and parametric maps, that are both amenable for display and quantitatively more accurate than what a post-reconstruction fitting and unconstrained direct reconstruction can achieve (low bias and low variance, Fig.3).
This method appears to be promising as a **feasible approach to applying kinetic modeling to very large 4D clinical datasets** with a ***reduced computational cost***, thanks to the parallel GPU implementation based on the analytic expression of the kinetic model and its derivatives.
Future studies will extend the current open-source implementation, by integrating different kinetic models (linear and non-linear) and different priors.
We have already started working to adapt the proposed KCS algorithm to deal with **DCE-MRI** (see Fig. 5) and **dynamic CT** tracer kinetic modeling.

References

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